MATHEMATICAL MODEL OF MORNING GLORY SPILLWAYS USING ARTIFICIAL NEURAL NETWORKS.

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ABSTRACT

Relationships between the discharge coefficient (C) and the ratio of head on and radius of the morning -glory spillway have been determined by means of hydraulic models (Wagner W.E. 1956; USBR, 1987) for three conditions of approach flow depths (P). These experimental curves show that the discharge coefficient increases inversely with head on the crest, contrary to what happens for a straight edge overfall spillway. This coefficient differs from a straight edge crest due to the effects of the submersion and the convergence of the flow lines toward the center of the funnel and the existence of vortices. A precise value of discharge coefficient for relationships H/R and P/R not tested previously are needed for design and operation diagnosis. These values have to be obtained through laboratory tests in order to determine discharge rating curves under free-flowing and drowned conditions.

This paper uses artificial neural networks as a mathematical tool of interpolation and extrapolation of non-linear curves to encapsulate the universe of laboratory experiments carried out in hydraulic models. This technique has already been successful in other applications such as the computation of correction factors for Penman evaporation equation and also in generating designs storms.

The mathematical model was compared with experimental data. Results show that a trained neuronal network is able to interpolate and extrapolate correctly values of C for relationships H/R and P/R not previously tested in laboratory. Validation of the neural network was made comparing the results of the model with existing spillway observations: morning glory of lake San Roque and Dique Potrerillos in Argentina and with the hydraulic model of El Toro spillway in Chile.

Results indicate that an artificial neuronal network constitutes a valuable support tool in the design and diagnosis of operation of morning glory spillways, allowing to contrast the operation of spillways of any geometry with spillways of equal radio and approach depth. This capacity helps to diagnose discharge problems caused by defects in the profile form of the crest of the spillway.

KEYWORDS

Morning Glory spillway; Spillway

INTRODUCTION

Discharge of a morning glory spillway is a function of crest radius, diameter and length of outlet pipe, geometry of the crest and crest lip and other variables. (Figure 1).
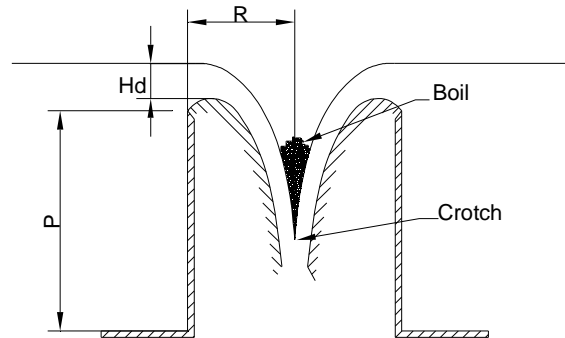
Figure 1.- Section of a Morning Glory weir.

The flow discharge can be computed using equation [1] (in British units) when the crest and form of the lower lip adjust to the trajectory of a free overfall:

$$Q \equiv C_0 \cdot L \cdot H_d^{3/2} \quad [1].$$

Where:

H_d : Head over the crest measured from the lower lip of the weir, assuming negligible the approach velocity
L : total length of weir ($2 \cdot \pi \cdot R$)
R : external radius of outlet pipe
C_0 : Discharge coefficient, function of H_d/R

Wagner (1956) performed laboratory experiments to produce empirical curves relating the discharge coefficient with H_d/R for different values of P/R (being P the height of the weir) Figure 2.

The discharge coefficient calculated using experimental results is realistic if the following conditions are met:

1. The weir crest and the form of the transition correspond to the trajectory of a thin circular weir with head equal to H_d.
2. The nappe is aerated so that no pressures below the atmospheric on the lower nappe are present.

Wagner performed laboratory experiments for values of P/R equal to 0.15, 0.3, 2.0. The curves show that the coefficient decreases when the head increase. This behavior is contrary of what is observed in rectangular weirs.
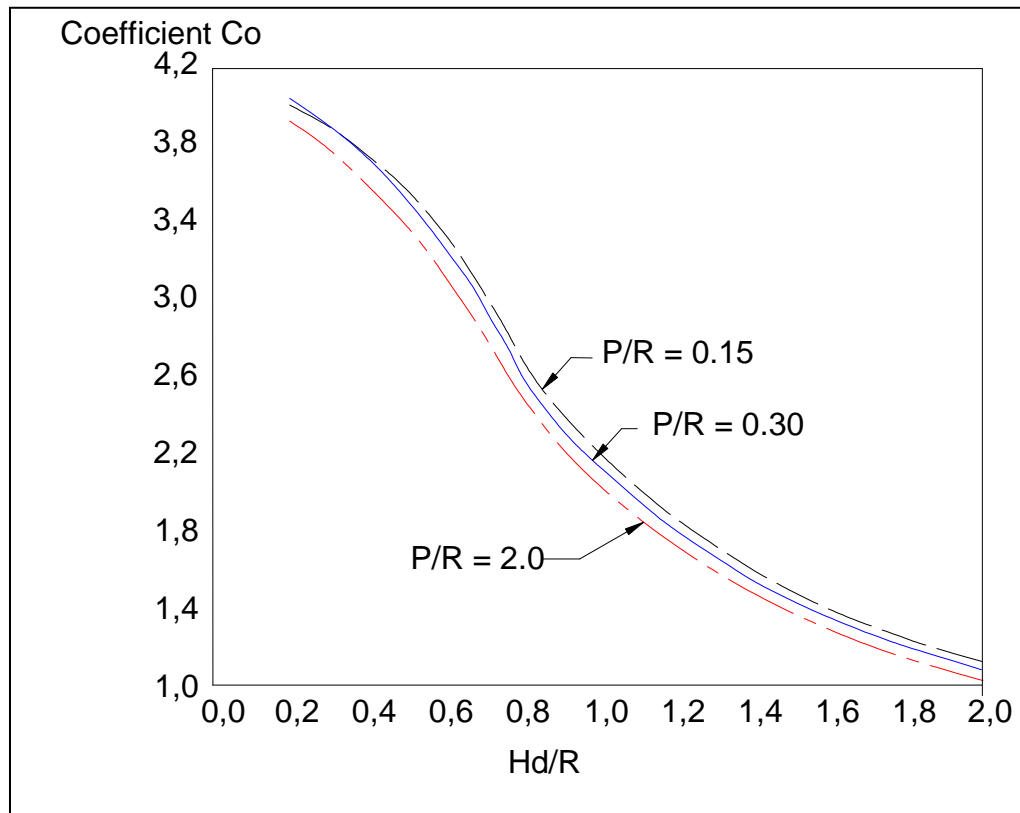
Figure 2.- Discharge coefficient C_0 versus H_d/R and P/R

ARTIFICIAL NEURAL NETWORK (ANN)

Wagner experiences, published by the U.S. Bureau of Reclamation (USBR, 1987), were used to formulate a neural network which would encapsulate the information of the discharge coefficient for different values of *P/R* and *Hd/R*. This model is able to interpolate and extrapolate laboratory results. This interpolation and extrapolation scheme using ANN was used in a different setting with good results. (Toro, Dölling and Varas, 2005).

An artificial neural network is a mathematical model formed by units called neurons and links or connections between neurons. Each neuron receives an input, which is the weighted sum of the outputs of other neurons. The neuron acts on this input by means of an activation function and a bias value. Activation functions are continuous and differentiable and can be linear or non linear. Neurons are arranged in layers. A network has an input layer, one or more hidden layers and an output layer. The weight assigned to each link is determined in such a way as to minimize the errors to reproduce the desirable result or output.

Wagner experimental results were captured with an ANN having 2 input neurons, 5 hidden neurons and one output neuron. A sigmoid activation function was used and weights were calculated using a back propagation algorithm. Model architecture can be summarized as an ANN 2-5-1. The final output is represented by equation [2]. The algorithm is detailed in Figure 3.

$$o_8 = \left(1 + e^{\left[-\theta_8 - \sum\limits_{j=3}^{7} \left[w_{j,8} \cdot \left[1+e^{\left(-\theta_j - \sum\limits_{i=1}^{2} w_{i,j} \cdot x_i \right)^{-1}} \right] \right] \right]} \right)^{-1}$$

[2]

Where

$X_i \quad i = 1, 2 \ \ldots\ldots$..input variables (2 input neurons)

$w_{i,j}$ = weight for the connection between neuron i and neuron j

$w_{i,k}$ = weight for the connection between neuron j and neuron k

$\theta_j \quad j = 3,4,5,6 \ y \ 7$ bias of neuron j

$\theta_8 =$ output neuron threshold

O_8 = output of ANN = calculated value of C_0

INPUT LAYER

$X_1 =$ explanatory variable 1 (H_d/R)

$X_2 =$ explanatory variable 2 (P/R)

HIDDEN LAYER

$y_j = \theta_j + \sum\limits_{i=1}^{2} w_{i,j} \cdot X_i \quad \forall j = 3..7$ activation value of hidden neuron j

$\sigma(y_j) = \dfrac{1}{1 + e^{-y_j}} \quad \forall j = 3..7$ output value of hidden neuron j

OUTPUT LAYER

$y_8 = \theta_8 + \sum\limits_{j=3}^{7} w_{j,8} \cdot \sigma(y_j)$ activation value of the output neuron 8 with bias

$\sigma(y_8) = \dfrac{1}{1 + e^{-y_8}}$ output value of neuron 8

$o_8 = \sigma(y_8) =$ Value of C_0 for the relationships H_d/R and P/R inputs as X_1 y X_2.

Figure 3.- Mathematical formulation of the neuronal network ANN 2-5-1 for Co calculation

The ANN was trained using 880 input-output sets. Both input and output were scaled to be in the range [0, 0.7] using equations 3a, 3b and 3c. Scaling is useful for extrapolation purposes.

Scaling equations:

$X_1 = (H_d/R) \cdot (0.7 / 2)$ [3-a]

$X_2 = P/R \cdot (0.7 / 2)$ [3-b]

$o_8 = \sigma(y_8) \cdot 0.7 / 4.0452$ [3-c]

The Stuttgart Neural Network software (SNNS)(Zell et al., 1995) was used to compute the weights and bias factors that minimize errors in the output variables. Best solution was reached after 25,000 cycles. Final weights and bias are presented in Tables 1 and 2.

**Table 1.-** Weights of the links between neurons.

INPUTS	HIDDEN LAYER				
	3	4	5	6	7
1	-0.14195	-0.00682	-3.78622	-0.53570	12.99177
2	0.55902	-0.70691	0.00093	-0.40084	0.16541
OUTPUT 8	-0.63498	-0.17337	2.40005	0.77311	-1.51658

Table 2.- Bias of the neurons

|	HIDDEN LAYER					
	3	4	5	6	7	8
θ	-0.88173	-0.48617	2.89108	-0.22619	-3.20135	-1.33786

MODEL VALIDATION.

Figure 4 compares model results with experimental values for $P/R = 0.15$ and $P/R = 2$.
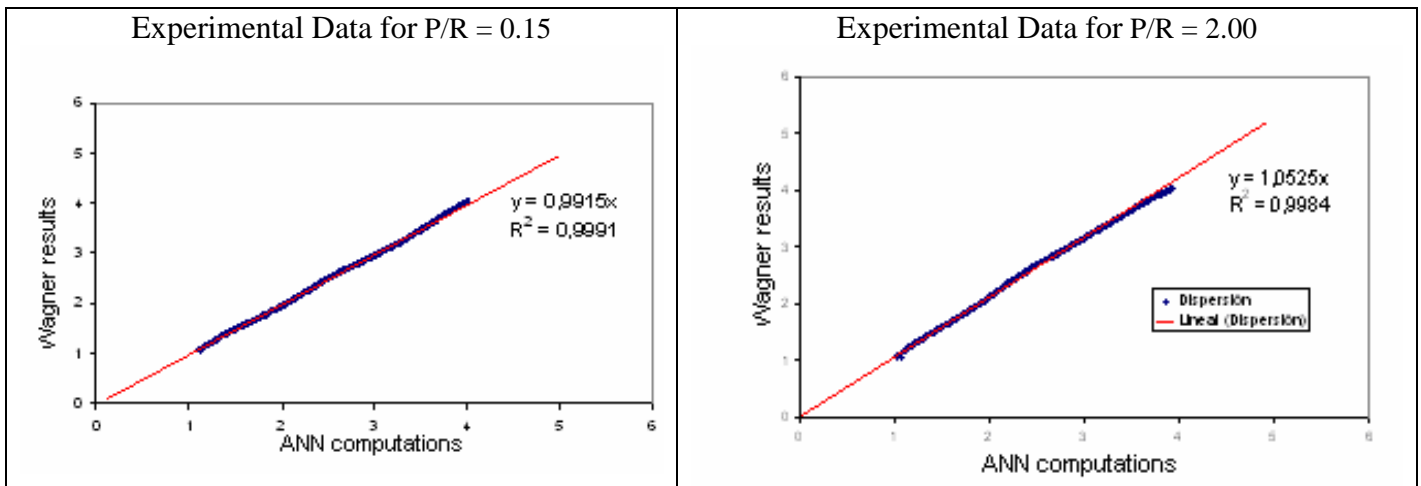
Figure 4.- Dispersion Diagrams. Wagner results and ANN 2-5-1 computations

Interpolation and extrapolation capabilities of the model are illustrated in Figure 5. It can be observed that computed curves follow closely the results presented by Wagner.
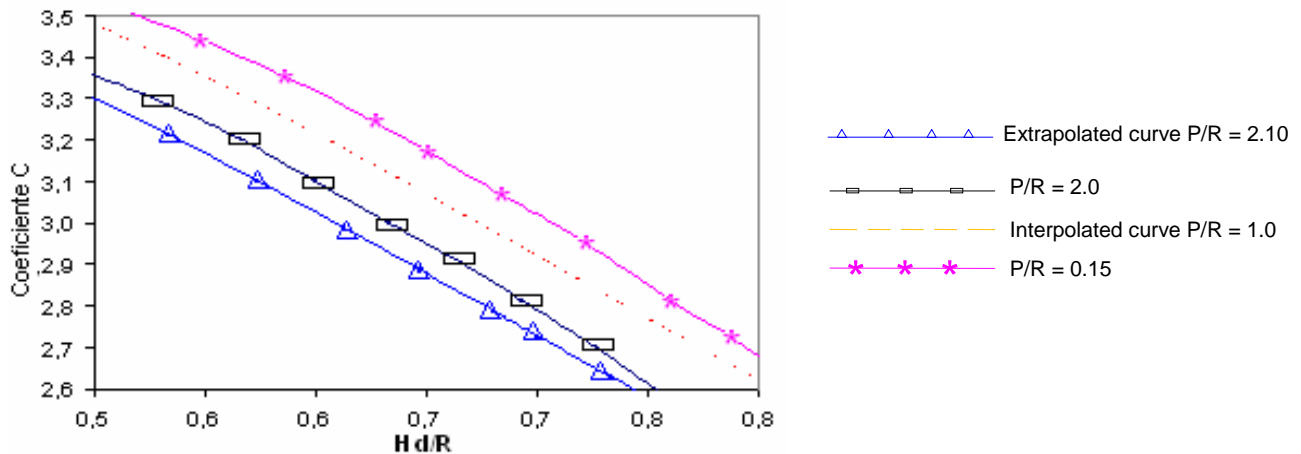
Figure 5.- Interpolated and extrapolated curves for C_0 using ANN 2-5-1.
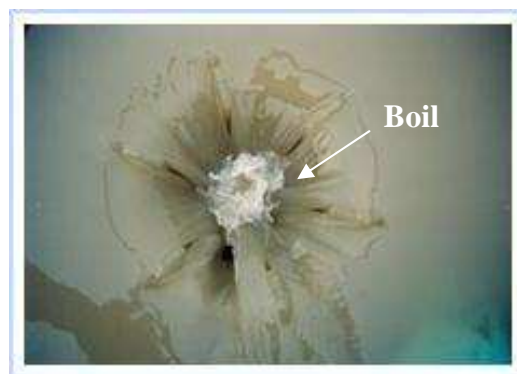
FLOW CONDITIONS

Free flow conditions prevail for head-radius ratios less than 0.45. As head increases partial submergence is presented and flow is controlled by submerged weir conditions. When Hd/R ratios approach unity the weir operates completely submerged and the flow is represented by an orifice flow condition. If head continues to increase the weir operates under pipe flow conditions.

The point where the annular nappe joins the solid jet is called the crotch. Once the flow forms a solid jet, a boil will form above the crotch. As the flow increases both the crotch and the boil will reach the surface and a slight depression and/or eddies will show at the surface. At this point the weir is drown out. Vortex action must be minimized so that the flow converges into the drop. Orifice flow is governed by equation [4] following the Torricelli principle. Discharge coefficient in this case takes into account energy losses due to the formation of vortices, changes in flow direction and friction. Discharge coefficient values will depend on the geometry of the weir, pipe and vortex characteristics.

$$Q = Cp \cdot \left(\pi \cdot \frac{D^2}{4} \right) \cdot \sqrt{2 \cdot g \cdot (hf + Hd)} \qquad [4]$$

$$Q = \frac{1}{n} \cdot A \cdot Rh^{2/3} \cdot \sqrt{i}; \quad A = (\theta - \sin(\theta)) \cdot \frac{D^2}{8}; \quad Pm = D \cdot \frac{\theta}{2}; \quad Rh = \frac{A}{Pm} \qquad [5]$$

Variables are defined in Table 4.

Figure 6.- Boil formation in Santa Juana hydraulic model for Q= 1200 m^3/seg.

To determine the point when the weir is drown out (Figure 6) a recursive algorithm was formulated. This algorithm compares the discharges under pipe flow conditions (equation 5) with orifice flow conditions (equation 4). When both values are equal the program continues to use equation 4 as head increases. In this case a coefficient C_p is introduced to take into account minor energy losses in the outlet pipe.

CASE STUDIES

Real hydraulic and physical data of morning glory weirs of San Roque dam in the Province of Cordoba, Argentina; Potrerillos dam in Mendoza, Argentina, and Santa Juana Dam, Chile, were collected in order to validate model behavior. Santa Juana dam finally did not include a Morning Glory weir as a discharge structure, so the information available corresponds to the hydraulic model.

San Roque Spillway

San Roque weir (Figure 7a) has a radius of 10.28 m, a height of 10 m (P), and a 4m diameter. The outlet pipe has a total head of 25 m, a 4 m diameter, a slope of 2.5% and a Manning roughness coefficient of 0.014.

The crest circumference is not complete due to the fact that the structure is anchored in a rock formation. This fact produces an additional energy loss, which must be taken into account in the discharge introducing a contraction coefficient.
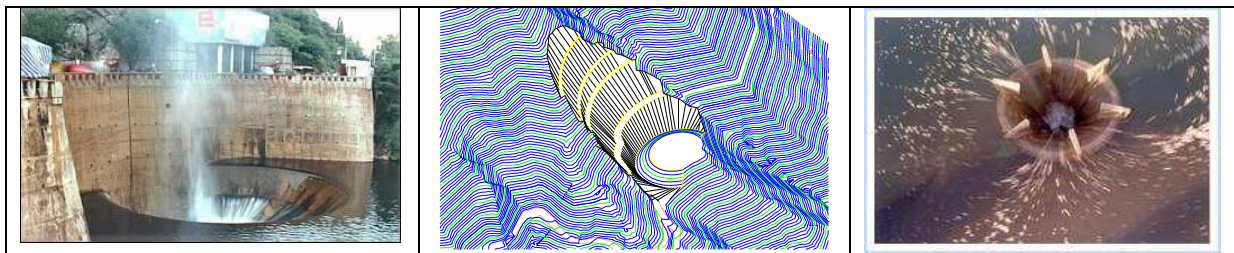
Figure 7.- Morning Glory weirs.: a) San Roque, b) Potrerillos y c)Santa Juana.

Potrerillos Spillway

Potrerillos weir (figure 7-b) has a crest radius R = 16.75m, weir height P = 15m and a total head in pipe outlet of hf = 65 m. Outlet pipe has a diameter of D = 12 m., a Manning roughness coefficient n = 0.014 and a slope i = 0.12 %. This weir has a complete crest circumference, however there is a rock outcrop close to it, which causes a change in streamflows which needs to be taken into account when modeling the flow.

Santa Juana Spillway

Physical hydraulic model studies were undertaken for Santa Juana dam (Fernández, 1989), (figure 7-c). This weir has a crest radius of R= 13.8 m, weir height P= 10.62 m. Outlet pipe has a total difference in elevation of 74.8 m a Manning roughness coefficient of n = 0.014 and a slope of 0.2%. The weir crest has a complete circumference and was tested with guiding vanes, which improve flow conditions but slightly diminish weir length.

EXTEND MODEL

The ANN model was implemented in EXTEND (Imagine That, 2002), (Figure 8). The program requires as input the geometric data of the weir and calculates de discharge coefficient and discharge for different values of the available head. It includes a module to transform units of measurement to SI system, a module to input the weir characteristics and a module to present the output graphically. Table 4 and Figure 9 show the details of the required input data and the dialog screens.
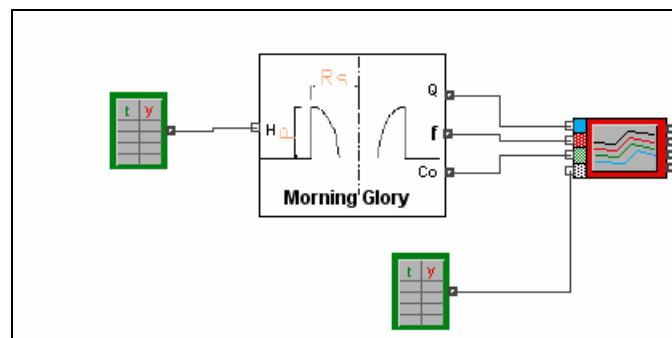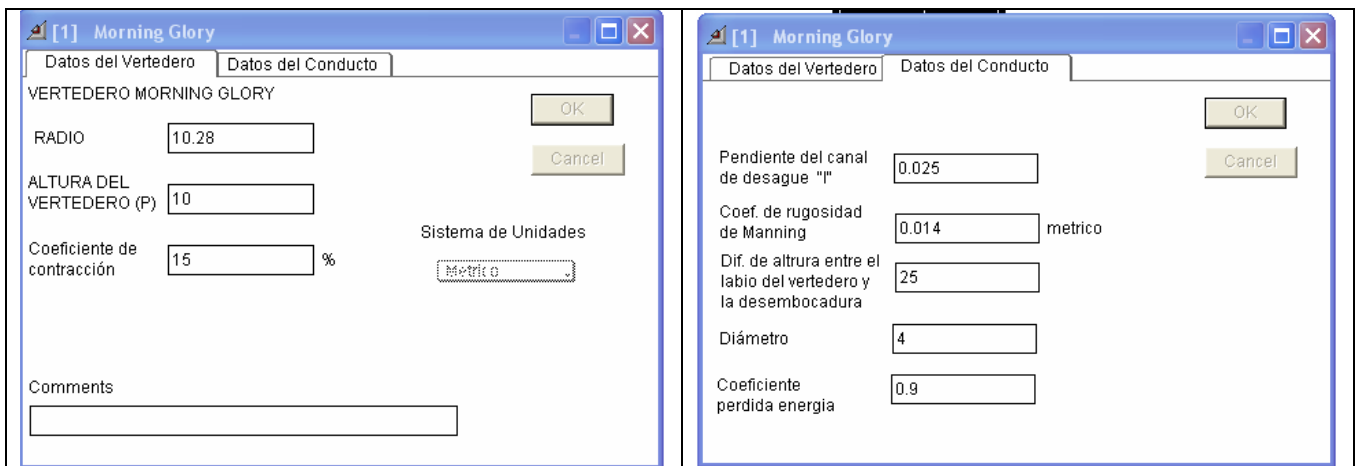

Figure 8.- Morning Glory EXTEND Model.

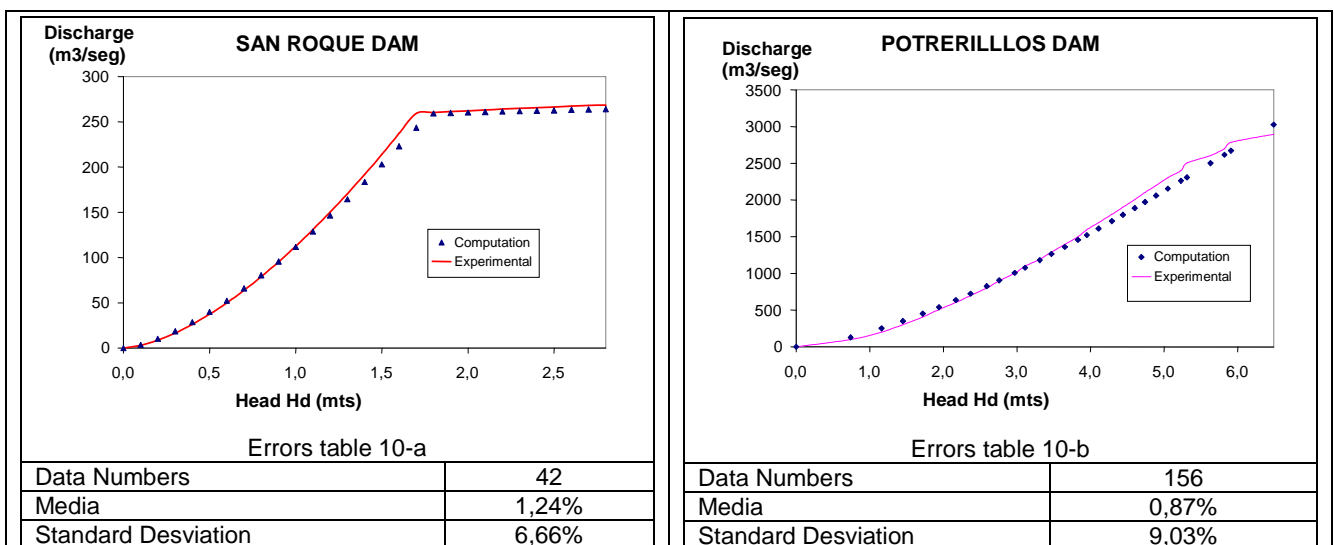Table 4 summarizes input data.

Table 4.- Input Data..

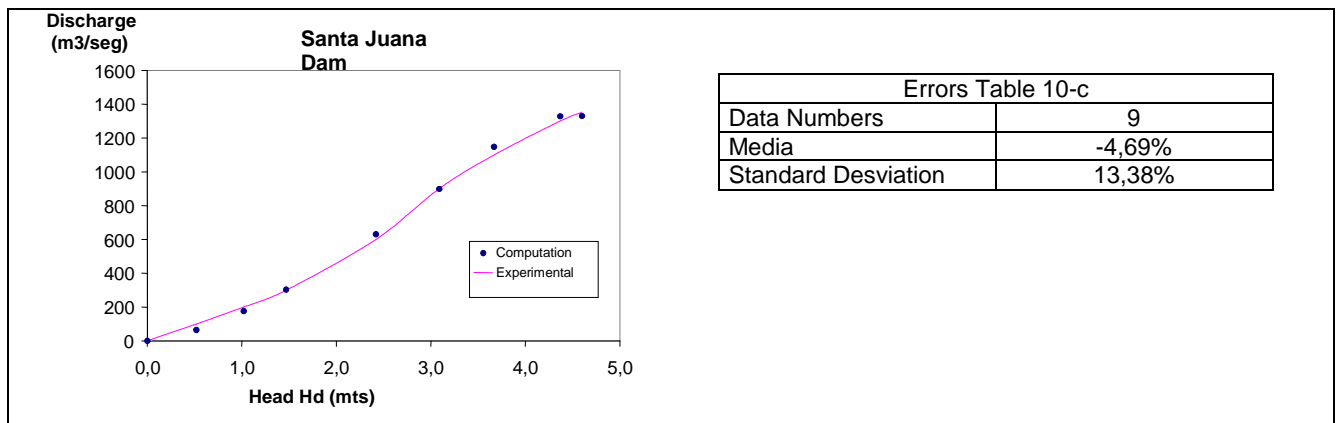| Variable	SI Units
Radius (R)	m
Weir height (P)	m
Contraction coefficient (Cc)	%
Energy Gradient (i)	M/m
Manning roughnesscoefficient (n)	Métric
Total head in pipe (hf)	m
Pipe diameter (D)	m
Pipe energy loss coefficient (Cp)	none

Figure 9.- Dialog screens of Morning Glory Block.

RESULTS

Figure 10 shows a comparison between observed and computed results for the three case studies, San Roque, Potrerillos and Santa Juana. The inflection point in the curves indicates the point where hydraulic behavior changes due to the drowning condition of the structure. The average and standard deviation of errors between computed and measured discharges is indicated in each graph.

Errors table 10-a

Data Numbers	42
Media	1,24%
Standard Desviation	6,66%

Errors table 10-b

Data Numbers	156
Media	0,87%
Standard Desviation	9,03%

Figure 10.- Experimental versus Computed Discharge a) San Roque Dam, b) Potrerillos Dam y c) Santa Juana Dam.

In the case of San Roque a 10% minor energy loss coefficient was used to represent losses due to aeration problems in the intake and bends in the outlet pipe. In some cases a vertical jet of water and air was observed, as shown in Figure 7-a. A contraction coefficient was also used to take into account the effect of the rock outcrop, which decrease the effective length of the weir crest.

Potrerillos Dam laboratory results do not include drowning conditions. A contraction coefficient of 15.5% was used to represent the effect of the rock that influences the development of a symmetrical streamflow pattern.

In Santa Juana the contraction coefficient takes into account the presence of guiding vanes, and the energy loss coefficient was 0.53. Laboratory experiments indicated that drowning conditions were observed for relatively small heads due to poor aeration conditions.

CONCLUSIONS

The study cases show that the ANN model represents the observed measurements quite closely. The model includes the possibility of considering contraction effects, additional energy losses and drowning conditions as head increases. Future work will address the mathematical modeling of energy loss phenomena in the intake of the weir and in the outlet pipe in order to improve the performance of the model to treat the drowning effects. The ANN model has shown to be a valuable support tool to analyze and represent flow conditions in morning glory type spillways.

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